Determining Material Properties and Beam Stresses Using Strain Gages

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Our goal in this laboratory was two-fold: to determine Young's modulus and Poisson's ratio of aluminum, and to characterize the principal stresses in a cantilevered aluminum beam. This was done using two different apparatuses. Using a constant stress beam with a transverse and an axial strain gage, the strains were measured and used to find the axial stress. From this, it was determined that Young's modulus was approximately 68.9 ± 1.0 GPa and Poisson's ratio was approximately 0.344 ± 0.027, both of which agreed or nearly agreed with published values. The second apparatus involved a constant cross-section beam with a strain gage rosette. Measuring strain once again, these values were used to find that the maximum and minimum principal stresses were 45.6 ± 1.4 MPa and 1.58 ± 1.9 MPa, respectively. Again, both agreed with that predicted by theory. In addition, the calculated angle of rotation, 31.3 ± 0.34°, agreed with the observed angle between the strain gage rosette and the central axis of the beam.

INTRODUCTION AND METHODS

In this investigation, we explored methods of measuring the properties of a material and the principal stresses acting on the material. To accomplish this, we performed two experiments. In the first experiment, we determined the Young's modulus and Poisson's ratio of a constant stress aluminum beam, as shown in Figure 1. Attached to the top of the beam were two strain gages, one aligned with the axial direction and one aligned with the transverse direction. When we were ready to begin taking measurements, we connected one of the strain gages to a Wheatstone bridge including two resistors and a potentiometer as shown in Figure 2. After the bridge was balanced using the potentiometer, weights were added incrementally to the hanger attached to the hook at the end of the beam. With each additional weight, the voltage across the bridge measured by the desktop digital multimeter was recorded.

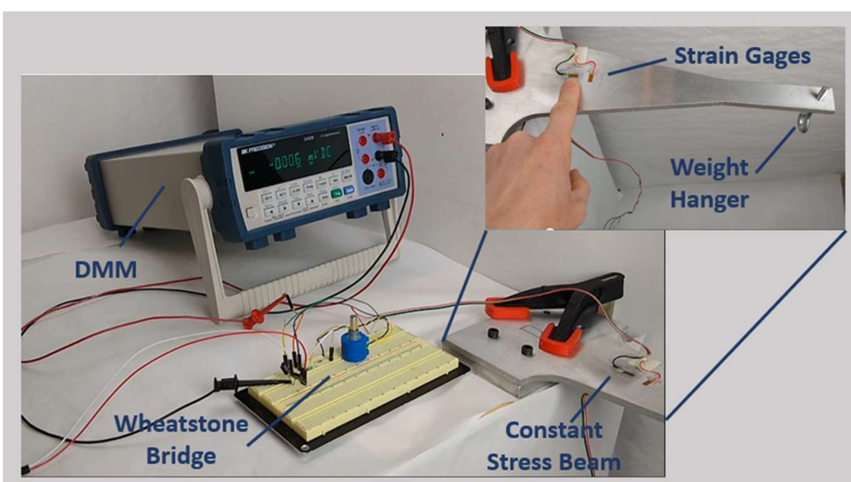


Figure 1: Experimental apparatus for measuring strain on a constant stress beam.

This data was then imported into MATLAB for analysis. Using Equation 1, the axial and transverse strains, ε_x and ε_y, were found from the voltage measurements, where the gage factor $F = 2.10 \pm 1\%$ [1].

$$\varepsilon = \frac{4\,\Delta V_{out}}{V_{in}\,F} \tag{1}$$

Next, we found the Poisson's ratio, υ, for our aluminum beam by curve fitting the strain data to the following equation using MATLAB's *lsqcurvefit* command [1]. To achieve an accurate fit, we needed to include a y-intercept (b).

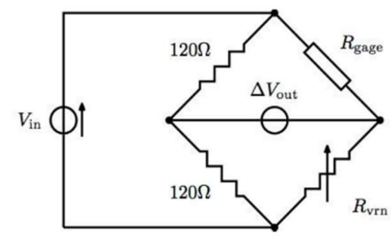


Figure 2: Each gage was connected to the Wheatstone bridge. [1]

$$-\varepsilon_y = \upsilon\,\varepsilon_x + b \tag{2}$$

As mentioned previously, the other property we cared about was Young's modulus, E, which can be found from the relationship between axial stress and strain [1]:

$$\sigma = E\,\varepsilon_x + b \tag{3}$$

where the theoretical axial stress σ is given by

$$\sigma = \frac{6Pd}{bh^2} \tag{4}$$

in which P is the load, d is the distance between the strain gage and the application of the load, and b and h are the width and thickness of the beam at the strain gage, respectively [1]. Again, using *lsqcurvefit* with a y-intercept to fit the axial stress and strain, Young's modulus could be calculated.

The purpose of the second experiment was to measure the principal stresses and angle of rotation in a constant cross-section beam. As shown in Figure 3, a cantilevered aluminum beam with a strain gage rosette attached to its top was mounted in a fixture containing a micrometer. As in the first experiment, we constructed a Wheatstone bridge containing one of the strain gages and balanced it using the potentiometer. After noting the position of the micrometer when there was no deflection, the micrometer deflected the beam by 0.4 inches. We recorded the resulting voltage displayed on the DMM. This procedure was repeated for the remaining two strain gages in the rosette.

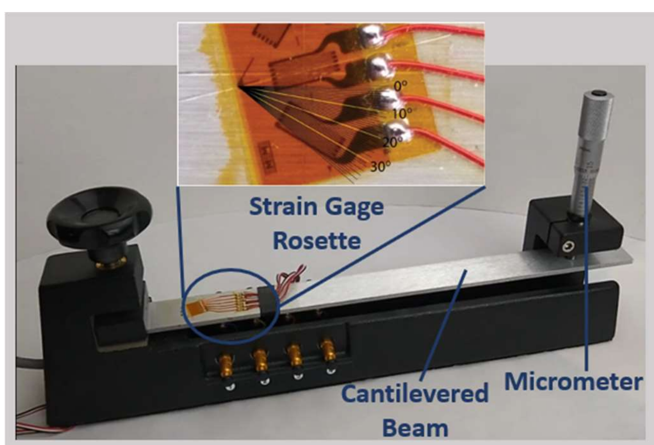


Figure 3: Apparatus used to determine the principal stresses, which included a cantilevered beam and a strain gage rosette.

As for the constant stress beam analysis, we used the voltage measurements from each of the three gages to find the strains ε_A, ε_B, and ε_C using Equation 1. From these, we could calculate the principle strains, which are given by:

$$\varepsilon_{max,min} = \frac{\varepsilon_A + \varepsilon_C}{2} \pm \sqrt{\frac{(\varepsilon_A - \varepsilon_B)^2 + (\varepsilon_C - \varepsilon_B)^2}{2}} \tag{5}$$

where, in our experimental apparatus, $\varepsilon_1 = \varepsilon_{max}$ and $\varepsilon_2 = \varepsilon_{min}$ [1].

From this, we could find the principal stresses acting in the beam. Using the values for Young's modulus and Poisson's ratio found in the first experiment, the maximum and minimum principal stresses are given by [1]:

$$\sigma_1 = \frac{E}{1 - \nu^2}(\varepsilon_1 + \nu\varepsilon_2) \tag{6}$$

$$\sigma_2 = \frac{E}{1 - \nu^2}(\varepsilon_2 + \nu\varepsilon_1) \tag{7}$$

The three measured strains were also used to determine the angle between the principal axes and the gage axis using the relationship [1]:

$$\theta_P = \frac{1}{2}\tan^{-1}\left(\frac{\varepsilon_A - 2\varepsilon_B + \varepsilon_C}{\varepsilon_A - \varepsilon_C}\right) \tag{8}$$

This value was compared to the physical orientation of the strain gage with relation to the central axis of the beam, which was measured using a protractor as seen in Figure 3.

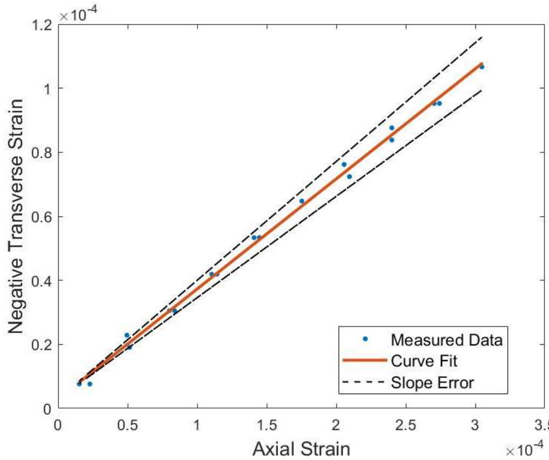
To compare our result for the maximum principal stress to the theoretical axial stress, which is the maximum principal stress in the beam, we calculated the theoretical stress due to a deflection δ using the following equation:

$$\sigma_{theory} = \frac{3E\delta d(\frac{h}{2})}{L^3} \tag{9}$$

where L is the distance between the application of the load and the base of the cantilevered beam, d is the distance between the load and the strain gage rosette, and h is the thickness of the beam. Since the maximum principal stress corresponds with the axial stress, then it follows that the minimum principal stress corresponds with the transverse stress. As a result, σ_2 should be zero for a bending load.

RESULTS AND DISCUSSION

As discussed previously, both Young's modulus and Poisson's ratio were determined by curve fitting the data from the constant stress beam to find the slopes, as shown in Figure 4.

a) 

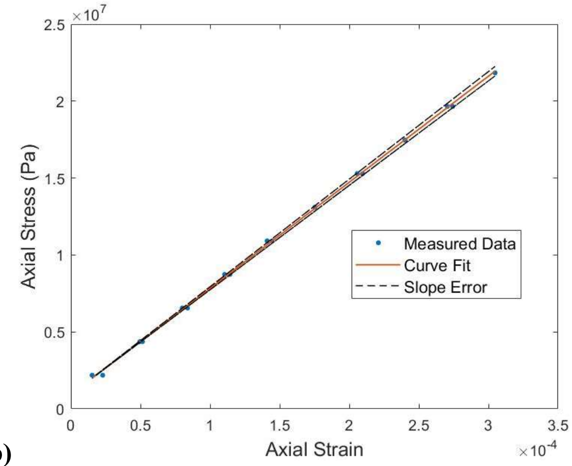
b) 

Figure 4: a) The relationship between transverse and axial strain were used to find Poisson's ratio. b) Similarly, the slope found from relating axial stress and strain provided Young's modulus.

Both plots show that the errors in the slopes from uncertainty propagation and curve fitting were relatively small. In addition, the data lies mostly within the error bounds, indicating that random errors were small as well. A linear fit appears to be a good representation of the relationship between the axial and transverse strains as well as the axial stress and strain, matching our theoretical understanding of these relationships.

The experimental values for Young's modulus and Poisson's ratio found using the constant stress beam are tabulated below, along with published values for aluminum.

Table 1: Comparison of experimental values for Poisson's ratio and Young's modulus to published values for aluminum.

Property	Experimental Value	Theoretical Value [2]
Poisson's Ratio, ν	0.344 ± 0.027	0.33
Young's Modulus, E (GPa)	68.9 ± 1.0	70

As seen in Table 1, the accepted value for Poisson's ratio falls within our experimental range, and thus we can conclude that our results agree with the theoretical value. However, the theoretical value for Young's modulus falls outside of our range. This small error may have come from the curve fit, as we needed to include fit a y-intercept in order to best fit our data. Because the percent error of the mean Young's modulus was only 1.6%, our result still appears to be a good approximation of this property. As a result, we concluded that our method of determining Poisson's ratio and Young's modulus was suitable.

These values were then used to calculate the principal stresses and the angle of rotation, which are presented in Table 2 along with their theoretical values.

Table 2: Comparison of experimental values for the principal stresses and angle of rotation to their theoretical values.

	Experimental Value	Theoretical Value
Maximum Prinicpal Stress (MPa)	45.6 ± 1.4	46.5 ± 1.2
Minimum Principal Stress (MPa)	1.58 ± 1.9	0
Angle of Rotation (degrees)	31.3 ± 0.34	30.5 ± 0.5

The experimental maximum principal stress, determined from the strain rosette, certainly agrees with the theoretical axial stress, calculated using our understanding of cantilevered beams under a load. In addition, while the mean minimum principal stress is greater than zero, the uncertainty is larger than the value itself, so it agrees with our hypothesis that there would be no transverse stress. Finally, the calculated angle of rotation agrees with the observed angle, indicating that we were correct in assuming that the maximum principal stress was aligned with the central axis of the beam.

CONCLUSION

This investigation was conducted to determine Poisson's ratio and Young's modulus of aluminum and to find the principal stresses in a cantilevered beam. These goals were accomplished using two different apparatuses: a constant stress beam to find the properties of aluminum and a constant cross-section beam to determine the principal stresses. It was found that Young's modulus was about 68.9 ± 1.0 GPa and Poisson's ratio was about 0.344 ± 0.027, both of which agreed or nearly agreed with published values. In addition, the maximum and minimum principal stresses were 45.6 ± 1.4 MPa and 1.58 ± 1.9 MPa, respectively. The maximum principal stress agreed with the theoretical axial stress, and the large uncertainty in the minimum principal stress meant that the value was essentially zero, matching the theoretical transverse stress. The correlation between the maximum and axial stresses was further supported by the agreement of the calculated angle of rotation, $31.3 \pm 0.34°$, with the observed angle between the strain gage rosette and the central axis of the beam. As a result, we can conclude that our experiments successfully characterized the aluminum and its loading.

REFERENCES

1. Sabatino, D. and Smith, J. Lafayette College. (2020). *Laboratory Project #5: Strain Measurements*. Easton, PA.
2. Massechussetts Institute of Technology. *Material: Aluminum.* Material Property Database. http://www.mit.edu/~6.777/matprops/aluminum.htm